

# Signatures of Accretion Disks in Quasar Microlensing

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## Abstract

We propose that relative variability on short time-scales of the multiple images of a lensed quasar, after removal of the time delay, may be caused by hot spots or other moving structures in the accretion disk crossing microlens caustics caused by stellar mass objects in the lensing galaxy. Such variability has been reported in the two images of 0957+561. The short durations would be due to the high rotation speed of the disk ( $v/c \sim 0.1$ ), rather than planetary mass objects in the slowly moving ( $v/c \sim 10^{-3}$ ) lens. This interpretation could be confirmed by finding periodicity, or correlations of the spectral and flux variations due to the Doppler effect in the disk. We also propose another signature of stationary accretion disks (with no intrinsic variability): the gradient of the magnification over the accretion disk should cause a relative color change between the images whose sign and amplitude are correlated with the time derivative of the flux difference between the images. Other color terms induced by the radial variation of disk colors are of second order in the magnification gradient. The methods proposed here can be used first to verify that accretion disks near supermassive black holes are the source of the continuum radiation from quasars, and then to study them.

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## 1. Introduction

For the past 30 years, the dominant view has been that the bulk of the luminosity of quasars originates in accretion disks spiraling in towards super-massive black holes (Lynden-Bell 1969; see Rees 1984, Blandford et al. 1990, Lin & Papaloizou 1996 for reviews). It is therefore striking that to date, there has been no clear observational proof of this hypothesis. The main features of the theory are: 1) The mass of the black hole is roughly estimated from its Eddington luminosity:  $M \gtrsim 10^8 M_\odot (L/10^{46} L_\odot)$ . 2) The temperature of the accretion disk is estimated from the “blue bump” in the continuum spectrum to be  $T \sim 5 \times 10^4 \text{K}$ . 3) The characteristic radius of the accretion disk is estimated assuming a roughly black body intensity and is  $\sim 10^3 \text{AU}$  for a bright quasar. The main observational difficulty is that the angular size of the accretion disk at a cosmological distance is then very small ( $\sim 1 \mu\text{as}$ ), and so has not been possible to resolve to date.

Microensing provides a potentially powerful probe of quasars since the Einstein ring of a stellar-mass lens is typically  $\sim 10^3 \text{AU}$ , i.e. of the same order as the expected size of the accretion disk. Microensing should occur in most lensed quasars. When multiple images of a quasar are produced by a lensing galaxy (with typical separations of  $1''$ ), the total surface density of the lens must be near the critical surface density. If a large fraction of this surface density is contributed by stars, then these stars will no longer cause microensing lightcurves as isolated point masses, but instead their caustic curves are linked in a complicated network. The lightcurve of a source moving behind a lens then results from the variation of the magnification of several “microimages” with separations of the order of the Einstein radii of the stars,  $\sim 1 \mu\text{as}$  (e.g., Wambsganss, Schneider, & Paczyński 1990), and is therefore very complex. Every time the source moves inside (outside) a caustic two new images appear (disappear), and the magnification reaches a maximum that depends on the size of the emitting region in the source.

Microensing was first detected by Irwin et al. (1989) in Huchra’s Lens (QSO 2237+0305; Huchra et al. 1985), and shortly thereafter by Schild & Smith (1990) in the original double quasar, 0957+561 (Walsh et al. 1979). To verify that photometric variation is due to microensing, it must be distinguished from other sources of variation, intrinsic variation in particular. If one of the macroimages is found to vary *relative* to the other, then one may conclude that one or the other image (or both) are being microensed. However, the two images arrive at the observer with a relative time delay. Thus, the flux ratio of the two images can be measured only by comparing observations that are separated by this delay and, of course, this can only be done after the time delay has been measured.

If the timescale of a microensing event is long compared to the time delay, the latter can be ignored to first approximation. This is the case for 2237+0305: the

time delays among the four images are known to be of order days from the model of the lens, even though they have not been measured. Since the microlensing events are of order months, the time delay is not important. Schild & Choffin (1986) first measured the time delay for 0957+561 at  $\tau = 1.1$  yr. Schild & Smith (1990) used this delay when they showed that the B image had brightened relative to the A image by about 0.2 mag over 10 years. This rise has subsequently slowed to a stop (R. Schild 1996, private communication).

Intensive monitoring programs were then carried out to measure the time-delay more precisely. Since the quasar varies on scales of one or two days at the few percent level, a precision measurement did not at first appear difficult. However, it was found by Schild (1990) that there does not exist *any* value for  $\tau$  that would cause the 1-day to 3-month variations in the A and B images to coincide. He and his collaborators concluded that the underlying cause of most of these short timescale, low amplitude ( $\Delta m \sim 0.02$ ) variations is microlensing, not intrinsic variability (Schild 1990; Schild & Smith 1991; Schild & Thomson 1995; Schild 1996). Kundić et al. (1996) also reported finding such variations in the course of measuring a more accurate time delay,  $\tau = 1.14 \pm 0.01$  yr.

The microlensing interpretation of the short timescale events proposed by Schild has not received wide acceptance principally because the short timescales would seem to imply that the dark matter in the lensing galaxy is almost entirely composed of “rogue planets”. The argument for the mass scale is simple. The characteristic physical Einstein radius is

$$r_E = \left( \frac{4GM D_{\text{OL}} D_{\text{LS}}}{c^2 D_{\text{OS}}} \right)^{1/2}, \quad (1.1)$$

where  $D_{\text{OL}}$ ,  $D_{\text{OS}}$ , and  $D_{\text{LS}}$  are the angular diameter distances between the observer, lens, and source. For  $M = 1 M_{\odot}$ , and adopting  $H_0 = 55 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $\Omega = 1$ ,  $r_E/D_{\text{OL}} = (6.2, 2.1) \mu\text{as}$  for the lenses 2237+0305 and 0957+561, respectively. The crossing time is  $t_E = r_E/v_t$ , where  $v_t$  is the transverse velocity:

$$\mathbf{v}_t = \frac{\mathbf{v}_l - \mathbf{v}_o}{1 + z_l} - \frac{D_{\text{OL}}}{D_{\text{OS}}} \frac{\mathbf{v}_s - \mathbf{v}_o}{1 + z_s}. \quad (1.2)$$

Here,  $\mathbf{v}_o$ ,  $\mathbf{v}_l$ , and  $\mathbf{v}_s$  are the transverse velocities of the observer, lens, and source, and  $z_l$  and  $z_s$  are the redshifts of the lens and source (Kayser, Refsdal, & Stabell 1986; this formula is valid for flat cosmological models, in general the observer’s velocity relative to the CMB frame must be multiplied by a factor that depends on the space curvature, see Miralda-Escudé 1991). For 2237+0305 these parameters are  $(z_l, z_s) = (0.039, 1.69)$  and  $(D_{\text{OL}}, D_{\text{OS}}, D_{\text{LS}}) = (0.037, 0.29, 0.28)cH_0^{-1}$ , and for

0957+561 they are  $(z_l, z_s) = (0.36, 1.41)$  and  $(D_{\text{OL}}, D_{\text{OS}}, D_{\text{LS}}) = (0.21, 0.30, 0.18)cH_0^{-1}$ . The Sun's motion relative to the cosmic microwave background is  $v_o \simeq 300 \text{ km s}^{-1}$ , and if the transverse velocities of the lens and the source are similar then the timescale to cross the Einstein radius for  $M = 1 M_\odot$  is  $t_E \simeq (10, 30) \text{ yr}$  for 2237+0305 and 0957+561, respectively. Thus, when Irwin et al. (1989) reported a microlensing event with a duration of 3 months in 2237+0305, the naive interpretation was that this had been caused by a planet with a Jupiter mass. For events with durations of a few days in 0957+561, Earth masses would be inferred for the lensing objects. Wambsganss et al. (1990) pointed out that when the microlensing optical depth is large, events with timescales much smaller than  $t_E$  are possible (caused, for example, by the passage of the source near a cusp catastrophe), so low-mass lenses are not automatically implied from the observation of just one event. However, the average density of caustic crossings should still not be larger than  $\sim 1$  per Einstein radius, so events on short timescales should be very rare unless planetary mass objects account for a large fraction of the lens surface density.

However, an additional difficulty with the planetary-mass interpretation of the short events is the small size implied for the source. With the expected transverse speeds given above, the quasar in 2237+0305 moves by only  $0.2 \mu\text{as}$  relative to the lensing galaxy over three months, corresponding to  $\sim 300 \text{ AU}$  at the redshift of the quasar. This small size already implies a rather high temperature for the disk emitting the continuum ( $T \simeq 10^5 \text{ K}$ ; Rauch & Blandford 1991, see also Jaroszynski, Wambsganss, & Paczyński 1992). For the shorter events in 0957+561, and with an expected angular velocity  $\sim 10$  times smaller than in 2237+0305 (due to the larger distance to the lens), the inferred size of the source would be highly implausible within the framework of current models.

In this paper, we propose that microlensing can be used to resolve the structure of accretion disks. In § 2 we examine the possibility that short timescale events, of the type reported by Schild (1996), could be produced by intrinsic variability in the accretion disk combined with the effects of microlensing. In § 3 we turn to the case of a disk with no intrinsic variability, and propose a method to demonstrate the existence of an accretion disk from the spectral changes in the different macroimages as microlensing events take place.

## 2. Short Timescale Variability

The main difficulty in explaining the short timescale events is the small proper motion expected, corresponding to the typical galaxy velocities generated by large-scale structure of  $\sim 300 \text{ km s}^{-1}$ . However, the orbital velocity of the accretion disk in the continuum emitting region should be much larger. For example, if the black hole mass is  $M = 10^8 M_\odot$ , the circular velocity is  $3 \times 10^4 \text{ km s}^{-1}$  at a radius of 100 AU, implying that the proper motion of any orbiting blob is (5, 40) times larger than the proper motion of the lensing galaxy in 2237+0305 and 0957+561 [from eq. (1.2)]. We therefore propose that the short events observed in 0957+561 (and possibly some of the variations in 2237+0305 as well) are caused by “spots” moving at the typical orbital velocities. Such spots might be caused by a variety of phenomena, including instabilities in the accretion disk that create “hot spots” which are in roughly circular orbits, gas clouds or stars that fall to and crash against the accretion disk, or blobs ejected by a jet near the black hole. They could even be “cold spots”, relatively confined subluminal regions like sunspots on the Sun.

If the magnification of each image were uniform over the whole accretion disk, then the intrinsic variability of the quasar caused by such spots would be the same in all the images after correction for the time delay, and therefore they would not be assigned to microlensing. But if the magnification varies over the region of the motion of a hot spot by  $\Delta A$  due to microlensing, and the hot spot contributes a fraction  $\Delta F$  to the total quasar flux  $F$ , then we expect a relative variation of  $(\Delta A/A)(\Delta F/F)$  of the flux in different images. The timescale of these short events should be shorter than or of order the dynamical time in the accretion disk; for a radius  $r = 100 \text{ AU}$  and velocity  $v/c \sim 0.1$ , this is about one month (which should be redshifted by a factor  $\sim 2.5$  for the two quasars we have mentioned). If the spot is a transient phenomenon (changing its flux  $\Delta F$  over a dynamical time), we should expect an intrinsic variation of  $\Delta F/F$ , so the intrinsic and relative variation should be correlated.

The largest variations in magnification will occur when a caustic transits the accretion disk. Two images of the accretion disk will then be merging on a critical line and will be highly magnified. Let us assume that the spot has a surface brightness higher by a factor  $(1 + f_s)$  compared to the average of the accretion disk ( $f_s > 0$  for hot spots and  $f_s < 0$  for cold spots). Then the radius of the spot is  $R_s = (\Delta F/(f_s F))^{1/2} r$ , where  $r$  is the radius of the disk. Since the maximum magnification on a fold catastrophe is proportional to  $R_s^{-1/2}$  (e.g., Schneider, Ehlers, & Falco 1992), the fractional contribution of a spot to the total flux can rise to  $(\Delta F/F)(r/R_s)^{1/2} = (|\Delta F|/F)^{3/4} |f_s|^{1/4}$ . For example, a spot from a region

contributing only 0.2% of the total light and with  $|f_s| \sim 1$  could cause fluctuations  $\sim 1\%$ .

For a spot that is either falling to or being ejected from the black hole, the caustic will be crossed once. But if a spot is in circular orbit and can survive for several orbits, then the caustic will be crossed repeatedly. An example of the lightcurve that may result from this is illustrated in Figure 1. The middle panel shows the lightcurve for a single hot spot in circular orbit. We assume that the caustic crossing the disk is on a sufficiently large scale, so that the magnification over the disk can be approximated as  $A = A_0 + K/x^{1/2}$ . There is a characteristic pattern of variations as the spot crosses the caustic: an “M-shaped” event is produced at every orbit of the spot. These events might therefore be confused with similar events when the motion is linear and two folds of the same closed caustic are crossed. The periodic events become shorter until they disappear (the time-reversed lightcurve is of course equally likely if the motion of the disk is reversed). Such a spot would be likely to have a different spectrum than the average for the quasar (it might, for example, be much closer to a blackbody). The spectrum is redshifted and blueshifted as the spot moves along the orbit (the radial velocity is shown in the lower panel for our example). There should then be a correlation between the spectrum and the magnification. In our example, the spot is always more blueshifted at the beginning of each event. This blueshift effect may be detectable even in the continuum spectrum of a spot, given the large velocities involved. Notice that such changes of the spectrum due to microlensing of spots can be separated from any other intrinsic changes of the spectrum of the entire disk, if one focuses on *relative* variations of the spectrum between the multiple images of the quasar after correcting for the time-delay.

Whether an orbiting spot can survive for several orbits depends on the process that generates it. A topologically confined spot, such as a cold spot due to a magnetic vortex, may persist for many orbital periods. Long lived spots may also be generated by gravitationally confined disturbances in the gaseous disk, and by the effects of compact objects orbiting inside the disk (such as black holes or other massive stars that may have fallen to the disk). On the other hand, hydrodynamical disturbances would expand in their sound crossing time and would then be sheared out into arcs by differential rotation. For example, if the ratio of the sound speed to the circular velocity is 0.002 (e.g.  $T = 5 \times 10^4$  K and  $v/c = 0.1$ ), a spot with orbital radius  $r$  should, after  $N$  periods, have expanded radially to a size  $0.01Nr$  due to expansion, and tangentially to a size  $0.1N^2r$  due to differential rotation. Therefore, such a spot cannot survive for more than a few periods.

In brief, spots can give rise to a variety of short timescale signals in the difference between the flux from two quasar images. The deviations in the microlensed

image minus the non-microlensed images can be both positive and negative and can contain repeating, non-repeating, and quasi-repeating signals.

### 3. Long Timescale Variability from a Steady Disk

The color of a microlensed accretion disk will in general be different from that of the disk in the absence of microlensing, due to differential magnification over the disk. If there are two or more macroimages of the disk, then this color shift can be determined by comparing the colors of the two images observed at two times separated by the measured time delay. The effect of microlensing can then be unambiguously distinguished from intrinsic color changes. Color differences between the images will arise if either or both are microlensed but for simplicity of discussion we will assume that one image is microlensed and the other is not.

Color changes are caused by two distinct effects of different orders. Both require that the magnification vary over the source. The variation can be either smooth or due to a caustic. We focus here on the case of smooth variations in the magnification because these are present generically.

The first order effect arises from a coupling of the linear color gradient across the accretion disk (due to the disk's rotation) with the gradient of the magnification. This effect is analogous to the line shift induced by microlensing of stars which has already been studied in some detail (Maoz & Gould 1994). In the stellar case, the stellar lines are broadened by rotation because one side of the star is rotating toward the observer and is blueshifted while the other side is redshifted. If, for example, the blueshifted side of the star is more highly magnified, the blue wing of the line will also be more highly magnified and the line centroid will shift toward the blue. For a star with projected rotation speed  $v \sin i$  and radius  $r$ , the shift is

$$\Delta v = \frac{\zeta}{4} r |\nabla \ln A| v \sin i \sin \gamma, \quad (3.1)$$

where  $\nabla \ln A$  is the logarithmic gradient of the magnification and  $\gamma$  is the angle between the magnification gradient and the projected axis of rotation. The factor  $\zeta$  depends on the details of the geometry. For a line emitted uniformly over the surface of a star,  $\zeta = 1$ , while for a ring of emission such as the Ca II line at 393 nm in giant stars (Loeb & Sasselov 1995),  $\zeta = 2$ .

The origin of the broad lines seen in virtually all quasars is a subject of considerable debate. From their width ( $v \sim 10^4 \text{ km s}^{-1}$ ), they must come from regions that are at least  $\sim GM/v^2 \sim 10^3 \text{ AU}$  from the center. If they come from the accretion disk (or from any other structure with organized motion on similar scales) then they will be subject to a line shift given by equation (3.1). While the exact

value of the shift will depend on the details of the geometry and will in addition be time dependent, the general order of the effect should be  $\sim v/4 \sim 2000 \text{ km s}^{-1}$ , assuming  $r|\nabla \ln A| \sim 1$ . Notice that for  $r = 10^3 \text{ AU}$ , the angular size is  $\sim 1 \mu\text{as}$ ; since the typical Einstein radius is only 1 to 5 times larger, a magnification gradient  $|\nabla \ln A| \sim 1/r$  should be common. Hence, there should be an observable shift at some times if the broad line region is indeed associated with the accretion disk. Schild & Smith (1991) measured the MgII line of the two images 0957+561 in two observations separated approximately by the time delay. No difference in the lines was detected although it is not clear that the data were of sufficient quality to see the predicted line shift. In any event, a definitive test of the accretion-disk origin of the broad lines would require measurements at multiple epochs.

Even if the accretion disk does not give rise to broad lines, a magnification gradient can still generate a color shift. In this case, the entire quasi-thermal spectrum (“blue bump”) may be loosely considered as a giant “emission line” with a fractional width of order unity. More concretely, consider a color formed from the ratio of fluxes at two broad-band wavelengths  $\lambda_B$  and  $\lambda_R$  with spectral slopes  $\alpha_B$  and  $\alpha_R$ . By a calculation analogous to the one of Maoz & Gould (1994), the first order color shift  $\Delta(B - R)_1$  is given by

$$\Delta(B - R)_1 = \frac{2.5}{\ln 10} |\nabla \ln A| \frac{\sin i \sin \gamma}{2} \frac{\langle rv(\alpha_B - \alpha_R) \rangle}{c}, \quad (3.2)$$

where  $\langle rv(\alpha_B - \alpha_R) \rangle$  is the intensity weighted mean over the disk profile, and  $\gamma$  is the angle between the minor axis of the projected disk and the gradient of the magnification. Thus, for  $|\alpha_B - \alpha_R| \sim 1$ ,  $\nabla \ln A \sim 10^{-3} \text{ AU}^{-1}$ , and characteristic disk radius  $\sim 300 \text{ AU}$ , one might typically expect color shifts of order  $\sim 1\%$ .

The second effect that causes color variations in microlensing on a moving stationary disk, which is of second order, is due to the radial color variation. This was investigated numerically by Wambsganss & Paczyński (1991). The color change can be written, to second order, as (see, e.g., Gould & Welch 1996)

$$\Delta(B - R)_2 \sim \frac{\Lambda_B - \Lambda_R}{8} (r \nabla \ln A)^2, \quad (3.3)$$

where  $r$  is the characteristic radius of the disk, and

$$\Lambda_B = \frac{2 \int_0^\infty dr' r'^3 S_B(r')}{r^2 \int_0^\infty dr' r' S_B(r')}, \quad (3.4)$$

is the second radial moment of the disk in the  $B$  band (with the same definition for the  $R$  band). If, for example,  $\Lambda_B - \Lambda_R \simeq 0.1$ , and  $(r \nabla \ln A) \sim 0.3$  (which we



would expect typically), this color term is still substantially smaller than the first order term given by equation (3.2). The first order term, which is proportional to  $\nabla \ln A$ , then dominates the relative color variations of the quasar images. The time derivative of the magnification of the microlensed image is also proportional to  $\nabla \ln A$ :

$$\frac{d \ln A}{dt} = v_t |\nabla \ln A| \frac{D_{\text{OS}}}{D_{\text{OL}}} \frac{\cos \phi}{1 + z_l}, \quad (3.5)$$

where  $\phi$  is the angle between the magnification gradient and the transverse velocity. Hence the first order color term should be directly proportional to the time derivative of the magnification ratio (after taking out the time delay). This allows it to be distinguished from the second order term. In practice, the proportionality between the color and the magnification variation should not be perfect because, as the magnification varies, the angle  $\phi$  should also vary. Moreover, some of the color variations may come from a sufficiently large region in the disk so that the first-order term does not dominate. The maximum scale at which a first-order expansion of the magnification can be applied is reduced, of course, if low-mass stars or brown dwarfs are highly abundant, and is also highly variable in the magnification patterns generated by the random superposition of stars. Nevertheless, a correlation between the color and magnification variation should clearly be present. Detailed numerical simulations of lightcurves, with realistic disk models around Kerr black holes (e.g., Jaroszynski et al. 1992) should help determine the signatures of accretion disks that can be found.

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## FIGURE CAPTIONS

- 1) A representative example of the lightcurve that may result from an orbiting hot spot. The upper panel shows the trajectory of the spot over one orbit. The thick line represents the caustic (which is moving upwards relative to the disk) and the dashed line is the line of nodes of the orbit. The magnification near the caustic is assumed to be  $A = 1 + (b/y)^{1/2}$ , where  $y$  is the distance to the caustic, and we choose the constant  $b = 2 \mu\text{as}$ , equal to the Einstein radius of a  $1 M_{\odot}$  star. The radius of the orbit is  $r = 100 \text{ AU}$ , inclined at an angle  $50^{\circ}$  to the line-of-sight, and the line of nodes has an inclination of  $40^{\circ}$  relative to the caustic. The orbital velocity of the spot is assumed to be  $v/c = 0.1$ , and the relative proper motion of the caustic and the disk is 40 times smaller than that of the hot spot. The solid line in the middle panel shows the magnification of the spot as a function of time, for a size of the spot equal to 0.1 times the radius of the orbit (this size determines the maximum magnification at each caustic crossing). The magnification can be measured from the observed flux variation relative to another, non-microlensed image after correction for the time-delay. The dotted line is the actual flux variation of the spot, which is affected by intrinsic variation due to Doppler effects when the spot is assumed to emit a constant flux in its rest-frame. The lower panel shows the radial velocity of the spot.

